UNIVERSITY OF SASKATCHEWAN
ELECTRICAL ENGINEERING 455.3

**Assignment Quiz 2**
**October 10, 2001**

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name:
Student Number:

1. Determine the output of the LTI system defined by

$$h[n] = 2^n u[-n-2],$$

if the input is given by

$$x[n] = 2u[n-2] - 3u[n-9].$$

$$y[n] = \underbrace{\sum_{k=-\infty}^{\infty} 2u[n-2]\,2^{n-k}u[k-n-2]}_{y_1} - \underbrace{\sum_{k=-\infty}^{\infty} 3u[n-9]\,2^{n-k}u[k-n-2]}_{y_2}$$

$y_1$:

$n \le 0$

$$y_1 = \sum_{k=2}^{\infty} 2\cdot 2^{n-k} = 2^{n+1}\left(\frac{(\frac{1}{2})^2 - \cancel{(\frac{1}{2})^{\infty}}\to 0}{1-\frac{1}{2}}\right) = 2^{n+1}(\tfrac{1}{2}) = 2^n$$

$n \ge 0$

$$y_1 = \sum_{k=n+2}^{\infty} 2^{n+1-k} = 2^{n+1}\left(\frac{(\frac{1}{2})^{n+2} - \cancel{(\frac{1}{2})^{\infty}}\to 0}{1-\frac{1}{2}}\right) = 2^{n+1}\left(\tfrac{1}{2}\right)^{n+1} = 1$$

$2^n = 1$ when $n = 0$

$y_2$:

$n \le 7$

$$y_2 = \sum_{k=9}^{\infty} 3\cdot 2^{n-k} = 3\cdot 2^n\left(\frac{(\frac{1}{2})^9 - \cancel{(\frac{1}{2})^{\infty}}\to 0}{1-\frac{1}{2}}\right) = 3\cdot 2^n\left(\tfrac{1}{2}\right)^8 = 3\cdot 2^{n-8}$$

$n \ge 7$

$$y_2 = \sum_{k=n+2}^{\infty} 3\cdot 2^{n-k} = 3\cdot 2^n\left(\frac{(\frac{1}{2})^{n+2} - \cancel{(\frac{1}{2})^{\infty}}\to 0}{1-\frac{1}{2}}\right) = 3\cdot \cancel{2^n}\cdot\cancel{(\tfrac{1}{2})^n}\left(\tfrac{1}{2}\right)^2\cdot \cancel{2} = \frac{3}{2}$$

$3\cdot 2^{n-8} = \frac{3}{2}$

$2^{n-7} = 1$ $n=7$ critical point

$y = y_1 - y_2$

$$y[n] = \begin{cases} 2^n - 3\cdot 2^{n-8} & n \le 0 \\ 1 - 3\cdot 2^{n-8} & n \le 7 \\ -\frac{1}{2} & n \ge 7 \end{cases}$$

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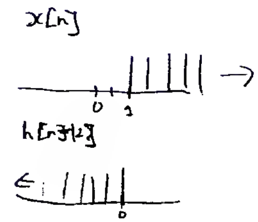
$$\sum_{k=N_1} \alpha^k = \frac{\alpha \quad \alpha}{1-\alpha}$$

1. Analytically determine the following discrete-time convolution.

$$y[n] = \alpha^n u[n] * \beta^n u[n-2], \quad |\alpha| < 1, |\beta| < 1$$

$h[n]$ $x[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^{n-k}u[n-k]\,\beta^k u[k-2]$$



$n < 2$ $n \ge 2$

$n-k > 0$ $k-2>0$
$n > k$ $k > 2$

$n<2$

$y[n] = 0$

$n \ge 2$

$$\sum_{k=2}^{n} \alpha^{n-k}\beta^k$$

$$\alpha^n \sum_{k=2}^{n} (\alpha^{-1}\beta)^k$$

$$\alpha^n\left(\frac{(\alpha^{-1}\beta)^2 - (\alpha^{-1}\beta)^{n+1}}{1-(\alpha^{-1}\beta)}\right)$$

$$\alpha^n\left(\frac{(\alpha^{-2}\beta^2) - (\alpha^{-n-1}\beta^{n+1})}{1-(\alpha^{-1}\beta)}\right)$$

$$y[n] = \frac{\alpha^{n-2}\beta^2 - \alpha^{-1}\beta^{n+1}}{1-(\alpha^{-1}\beta)} = \frac{\alpha^n(\frac{\beta}{\alpha})^2 - \left(\frac{\beta^{n+1}}{\alpha}\right)}{1-\frac{\beta}{\alpha}}$$